LAPLACE OPERATORS AND DIFFUSIONS IN TANGENT BUNDLES OVER POISSON SPACES

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Abstract

Spaces of differential forms over configuration spaces with Poisson measures are constructed. The corresponding Laplacians (of Bochner and de Rham type) on 1-forms and associated semigroups are considered. Their probabilistic interpretation is given.

1 Introduction

Stochastic differential geometry of infinite-dimensional manifolds has been a very active topic of research in recent times. One of the important and intriguing problems discussed concerns the construction of spaces of differential forms over such manifolds and the study of the corresponding Laplace operators and associated (stochastic) cohomologies. A central role in this framework is played by the concept of the Dirichlet operator of a differentiable measure, which is actually an infinite-dimensional generalization of the Laplace–Beltrami operator on functions, respectively the Laplace–Witten–de Rham operator on differential forms. The study of the latter operator and the associated semigroup on finite-dimensional manifolds was the subject of many works, and it leads to deep results on the border of stochastic analysis, differential geometry and topology, and mathematical physics, see, e.g., [22], [19], [23]. Dirichlet forms on Clifford algebras were considered in [26]. In an infinite-dimensional situation, such questions were discussed in the flat case in [11], [12], [13], [4]. A regularized heat semigroup on differential forms over the infinite-dimensional torus was studied in [15]. A study of such questions on general infinite product manifolds was given in [2], [3]. The case of loop spaces was considered in [28], [30].

At the same time, there is a growing interest in geometry and analysis on Poisson spaces, i.e., on spaces of locally finite configurations in non-compact manifolds, equipped with Poisson measures. In [5], [6], [7], an approach to these spaces as to infinite-dimensional manifolds was initiated. This approach was motivated by the connection of such spaces with the theory of representations of diffeomorphism groups, see [25], [36], [27] (these references and [7], [9] also contain discussion of

relations with quantum physics). We refer the reader to [8], [9], [35], and references therein for further discussion of analysis on Poisson spaces and applications.

In the present work, we develop this point of view. We define spaces of differential forms over Poisson spaces. Next, we define and study Laplace operators acting in the spaces of 1-forms. We show, in particular, that the corresponding de Rham Laplacian can be expressed in terms of the Dirichlet operator on functions on the Poisson space and the Witten Laplacian on the initial manifold associated with the intensity of the corresponding Poisson measure. We give a probabilistic interpretation and investigate some properties of the associated semigroups. Let us remark that the study of Laplacians on n-forms by our methods is also possible, but it leads to more complicated constructions. It will be given in a forthcoming paper. The main general aim of our approach is to develop a framework which extends to Poisson spaces (as infinite-dimensional manifolds) the finite-dimensional Hodge–de Rham theory.

A different approach to the construction of differential forms and related objects over Poisson spaces, based on the "transfer principle" from Wiener spaces, is proposed in [34], see also [32] and [33].

2 Differential forms over configuration spaces

The aim of this section is to define differential forms over configuration spaces (as infinite-dimensional manifold). First, we recall some known facts and definitions concerning "manifold-like" structures and functional calculus on these spaces.

2.1 Functional calculus on configuration spaces

Our presentation in this subsection is based on [7], however for later use in the present paper we give a different description of some objects and results occurring in [7].

Let X be a complete, connected, oriented, C^∞ (non-compact) Riemannian manifold of dimension d. We denote by $\langle \bullet, \bullet \rangle_x$ the corresponding inner product in the tangent space T_xX to X at a point $x \in X$. The associated norm will be denoted by $| \bullet |_x$. Let also ∇^X stand for the gradient on X.

The configuration space Γ_X over X is defined as the set of all locally finite subsets (configurations) in X:

$$\Gamma_X := \{ \gamma \subset X \mid |\gamma \cap \Lambda| < \infty \text{ for each compact } \Lambda \subset X \}.$$

Here, $|A|$ denotes the cardinality of the set A.

We can identify any $\gamma \in \Gamma_X$ with the positive integer-valued Radon measure

$$\sum_{x\in\gamma} \varepsilon_x \subset \mathcal{M}(X),$$

where ε_x is the Dirac measure with mass at x, $\sum_{x\in\varnothing}\varepsilon_x$:=zero measure, and $\mathcal{M}(X)$ denotes the set of all positive Radon measures on the Borel σ-algebra $\mathcal{B}(X)$. The space Γ_X is endowed with the relative topology as a subset of the space $\mathcal{M}(X)$ with the vague topology, i.e., the weakest topology on Γ_X such that all maps

$$\Gamma_X \ni \gamma \mapsto \langle f, \gamma\rangle := \int_X f(x)\,\gamma(dx) \equiv \sum_{x\in\gamma} f(x)$$

are continuous. Here, $f \in C_0(X)$(:=the set of all continuous functions on X with compact support). Let $\mathcal{B}(\Gamma_X)$ denote the corresponding Borel σ-algebra.

Following [7], we define the tangent space to Γ_X at a point γ as the Hilbert space

$$T_\gamma\Gamma_X := L^2(X \to TX; d\gamma),$$

or equivalently

$$T_\gamma\Gamma_X = \bigoplus_{x\in\gamma} T_xX. \tag{1}$$

The scalar product and the norm in $T_\gamma\Gamma_X$ will be denoted by $\langle \bullet, \bullet\rangle_\gamma$ and $\|\bullet\|_\gamma$, respectively. Thus, each $V(\gamma) \in T_\gamma\Gamma_X$ has the form $V(\gamma) = (V(\gamma)_x)_{x\in\gamma}$, where $V(\gamma)_x \in T_xX$, and

$$\|V(\gamma)\|_\gamma^2 = \sum_{x\in\gamma} |V(\gamma)_x|_x^2.$$

Let $\gamma \in \Gamma_X$ and $x \in \gamma$. By $\mathcal{O}_{\gamma,x}$ we will denote an arbitrary open neighborhood of x in X such that the intersection of the closure of $\mathcal{O}_{\gamma,x}$ in X with $\gamma \setminus \{x\}$ is the empty set. For any fixed finite subconfiguration $\{x_1, \dots, x_k\} \subset \gamma$, we will always consider open neighborhoods $\mathcal{O}_{\gamma,x_1}, \dots, \mathcal{O}_{\gamma,x_k}$ with disjoint closures.

Now, for a measurable function $F\colon \Gamma_X \to \mathbb{R}$, $\gamma \in \Gamma_X$, and $\{x_1, \dots, x_k\} \subset \gamma$, we define a function $F_{x_1,\dots,x_k}(\gamma, \bullet)\colon \mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k} \to \mathbb{R}$ by

$$\begin{aligned}\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k} \ni (y_1, \dots, y_k) \mapsto F_{x_1,\dots,x_k}(\gamma, y_1, \dots, y_k) :=\\ = F((\gamma \setminus \{x_1, \dots, x_k\}) \cup \{y_1, \dots, y_k\}) \in \mathbb{R}.\end{aligned}$$

Since we will be interested only in the local behavior of the function $F_{x_1,\dots,x_k}(\gamma, \bullet)$ around the point $(x_1, \dots, x_k)$, we will not write explicitly which neighborhoods $\mathcal{O}_{\gamma,x_i}$ we use.

Definition 1 We say that a function $F : \Gamma_X \to \mathbb{R}^1$ is differentiable at $\gamma \in \Gamma_X$ if for each $x \in \gamma$ the function $F_x(\gamma, \cdot)$ is differentiable at x and

$$\nabla^\Gamma F(\gamma) = (\nabla^\Gamma F(\gamma)_x)_{x\in\gamma} \in T_\gamma\Gamma_X,$$

where

$$\nabla^\Gamma F(\gamma)_x := \nabla^X F_x(\gamma, x).$$

We will call $\nabla^\Gamma F(\gamma)$ the gradient of F at γ.

For a function F differentiable at γ and a vector $V(\gamma) \in T_\gamma \Gamma_X$, the directional derivative of F at the point γ along $V(\gamma)$ is defined by

$$\nabla_V^\Gamma F(\gamma) := \langle \nabla^\Gamma F(\gamma), V(\gamma) \rangle_\gamma.$$

In what follows, we will also use the shorthand notation

$$\nabla_x^X F(\gamma) := \nabla^X F_x(\gamma, x), \tag{2}$$

so that $\nabla^\Gamma F(\gamma) = (\nabla_x^X F(\gamma))_{x\in\gamma}$. It is easy to see that the operation ∇^Γ satisfies the usual properties of differentiation, including the Leibniz rule.

We define a class $\mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X)$ of smooth cylinder functions on Γ_X as follows:

Definition 2 A measurable bounded function $F : \Gamma_X \to \mathbb{R}^1$ belongs to $\mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X)$ iff:

(i) there exists a compact $\Lambda \subset X$ such that $F(\gamma) = F(\gamma_\Lambda)$ for all $\gamma \in \Gamma_X$, where $\gamma_\Lambda := \gamma \cap \Lambda$;

(ii) for any $\gamma \in \Gamma_X$ and $\{x_1, \dots, x_k\} \subset \gamma$, $k \in \mathbb{N}$, the function $F_{x_1,\dots,x_k}(\gamma, \bullet)$ is infinitely differentiable with derivatives uniformly bounded in γ and $x_1, \dots, x_k$ (i.e., the majorizing constant depends only on the order of differentiation but not on the specific choice of $\gamma \in \Gamma_X$, $k \in \mathbb{N}$, and $\{x_1, \dots, x_k\} \subset \gamma$).

Let us note that, for $F \in \mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X)$, only a finite number of coordinates of $\nabla^\Gamma F(\gamma)$ are not equal to zero, and so $\nabla^\Gamma F(\gamma) \in T_\gamma \Gamma_X$. Thus, each $F \in \mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X)$ is differentiable at any point $\gamma \in \Gamma_X$ in the sense of Definition 1.

Remark 1 In [7], the authors introduced the class $\mathcal{F}C_{\mathrm{b}}^\infty(\mathcal{D}, \Gamma_X)$ of functions on Γ_X of the form

$$F(\gamma) = g_F(\langle \varphi_1, \gamma \rangle, \dots, \langle \varphi_N, \gamma \rangle), \tag{3}$$

where $g_F \in C_{\mathrm{b}}^\infty(\mathbb{R}^N)$ and $\varphi_1, \dots, \varphi_N \in \mathcal{D} := C_0^\infty(X)$(:= the set of all C^∞-functions on X with compact support). Evidently, we have the inclusion

$$\mathcal{F}C_{\mathrm{b}}^\infty(\mathcal{D}, \Gamma_X) \subset \mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X),$$

and moreover, the gradient of F of the form (3) in the sense of Definition 1,

$$\nabla^\Gamma F(\gamma)_x = \sum_{i=1}^N \frac{\partial g_F}{\partial s_i}(\langle \varphi_1, \gamma \rangle, \dots, \langle \varphi_N, \gamma \rangle) \nabla^X \varphi_i(x),$$

coincides with the gradient of this function in the sense of [7].

2.2 Tensor bundles and cylinder forms over configuration spaces

Our next aim is to introduce differential forms on Γ_X.

Vector fields and first order differential forms on Γ_X will be identified with sections of the bundle $T\Gamma_X$. Higher order differential forms will be identified with sections of tensor bundles $\wedge^n(T\Gamma_X)$ with fibers

$$\wedge^n(T_\gamma\Gamma_X) = \wedge^n(L^2(X \to TX; \gamma)),$$

where $\wedge^n(\mathcal{H})$ (or $\mathcal{H}^{\wedge n}$) stands for the n-th antisymmetric tensor power of a Hilbert space $\mathcal{H}$. In what follows, we will use different representations of this space. Because of (1), we have

$$\wedge^n(T_\gamma\Gamma_X) = \wedge^n\bigg(\bigoplus_{x\in\gamma} T_xX\bigg). \tag{4}$$

Let us introduce the factor space X^n/S_n, where S_n is the permutation group of $\{1,\dots,n\}$ which naturally acts on X^n:

$$\sigma(x_1,\dots,x_n) = (x_{\sigma(1)},\dots,x_{\sigma(n)}), \qquad \sigma\in S_n.$$

The space X^n/S_n consists of equivalence classes $[x_1,\dots,x_n]$ and we will denote by $[x_1,\dots,x_n]_d$ an equivalence class $[x_1,\dots,x_n]$ such that the equality $x_{i_1} = x_{i_2} = \dots = x_{i_k}$ can hold only for $k \le d$ points. (In other words, any equivalence class $[x_1,\dots,x_n]$ is a multiple configuration in X, while $[x_1,\dots,x_n]_d$ is a multiple configuration with multiplicity of points $\le d$.) We will omit the lower index d in the case where $n \le d$. In what follows, instead of writing $[x_1,\dots,x_n]_d : \{x_1,\dots,x_n\} \subset \gamma$, we will use the shortened notation $[x_1,\dots,x_n]_d \subset \gamma$, though $[x_1,\dots,x_n]_d$ is not, of course, a set. We then have from (4):

$$\wedge^n(T_\gamma\Gamma_X) = \bigoplus_{[x_1,\dots,x_n]_d\subset\gamma} T_{x_1}X \wedge T_{x_2}X \wedge \dots \wedge T_{x_n}X, \tag{5}$$

since for each $\sigma\in S_n$ the spaces $T_{x_1}X \wedge T_{x_2}X \wedge \dots \wedge T_{x_n}X$ and $T_{x_{\sigma(1)}}X \wedge T_{x_{\sigma(2)}}X \wedge \dots \wedge T_{x_{\sigma(n)}}X$ coincide.

Thus, under a differential form ω of order n, $n\in\mathbb{N}$, over Γ_X, we will understand the mapping

$$\Gamma_X \ni \gamma \mapsto \omega(\gamma) \in \wedge^n(T_\gamma\Gamma_X).$$

We denote by $\omega(\gamma)_{[x_1,\dots,x_n]_d}$ the corresponding component in the decomposition (5).

In particular, in the case $n=1$, a 1-form V over Γ_X is given by the mapping

$$\Gamma_X \ni \gamma \mapsto V(\gamma) = (V(\gamma)_x)_{x\in\gamma} \in T_\gamma\Gamma_X.$$

For fixed $\gamma \in \Gamma_X$ and $x \in \gamma$, we consider the mapping

$$\mathcal{O}_{\gamma,x} \ni y \mapsto \omega_x(\gamma, y) := \omega(\gamma_y) \in \wedge^n(T_{\gamma_y}\Gamma_X),$$

where $\gamma_y := (\gamma \setminus \{x\}) \cup \{y\}$, which is a section of the Hilbert bundle

$$\wedge^n(T_{\gamma_y}\Gamma_X) \mapsto y \in \mathcal{O}_{\gamma,x} \tag{6}$$

over $\mathcal{O}_{\gamma,x}$. The Levi–Civita connection on TX generates in a natural way a connection on this bundle. We denote by $\nabla^X_{\gamma,x}$ the corresponding covariant derivative, and use the notation

$$\nabla^X_x \omega(\gamma) := \nabla^X_{\gamma,x}\omega_x(\gamma, x) \in T_xX \otimes (\wedge^n(T_\gamma\Gamma_X))$$

if the section $\omega_x(\gamma, \cdot)$ is differentiable at x. Analogously, we denote by Δ^X_x the corresponding Bochner Laplacian associated with the volume measure m on $\mathcal{O}_{\gamma,x}$ (see subsec. 3.2 where the notion of Bochner Laplacian is recalled).

Similarly, for a fixed $\gamma \in \Gamma_X$ and $\{x_1, \dots, x_n\} \subset \gamma$, we define a mapping

$$\begin{aligned}\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_n} \ni (y_1, \dots, y_n) \mapsto \omega_{x_1,\dots,x_n}(\gamma, y_1, \dots, y_n) &:= \\ = \omega(\gamma_{y_1,\dots,y_n}) \in \wedge^n(T_{\gamma_{y_1,\dots,y_n}}\Gamma_X),&\end{aligned}$$

where $\gamma_{y_1,\dots,y_n} := (\gamma \setminus \{x_1, \dots, x_n\}) \cup \{y_1, \dots, y_n\}$, which is a section of the Hilbert bundle

$$\wedge^n(T_{\gamma_{y_1,\dots,y_n}}\Gamma_X) \mapsto (y_1, \dots, y_n) \in \mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_n} \tag{7}$$

over $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_n}$.

Let us remark that, for any $\eta \subset \gamma$, the space $\wedge^n(T_\eta\Gamma_X)$ can be identified in a natural way with a subspace of $\wedge^n(T_\gamma\Gamma_X)$. In this sense, we will use expressions of the type $\omega(\gamma) = \omega(\eta)$ without additional explanations.

A set $\mathcal{F}\Omega^n$ of smooth cylinder n-forms over Γ_X will be defined as follows.

Definition 3 $\mathcal{F}\Omega^n$ is the set of n-forms ω over Γ_X which satisfy the following conditions:

(i) there exists a compact $\Lambda = \Lambda(\omega) \subset X$ such that $\omega(\gamma) = \omega(\gamma_\Lambda)$;

(ii) for each $\gamma \in \Gamma_X$ and $\{x_1, ..., x_n\} \subset \gamma$, the section $\omega_{x_1,...,x_n}(\gamma, \bullet)$ of the bundle (7) is infinitely differentiable at $(x_1, \dots, x_n)$, and bounded together with the derivatives uniformly in γ.

Remark 2 For each $\omega \in \mathcal{F}\Omega^n$, $\gamma \in \Gamma_X$, and any open bounded $\Lambda \supset \Lambda(\omega)$, we can define the form $\omega_{\Lambda,\gamma}$ on $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_n}$ by

$$\omega_{\Lambda,\gamma}(y_1, \dots, y_n) = \mathrm{Proj}_{\wedge^n(T_{y_1}X\oplus\cdots\oplus T_{y_n}X)}\, \omega(\gamma \setminus \{x_1, \dots, x_n\} \cup \{y_1, \dots, y_n\}), \tag{8}$$

where $\{x_1, \dots, x_n\} = \gamma \cap \Lambda$. The item (ii) of Definition 3 is obviously equivalent to the assumption $\omega_{\Lambda,\gamma}$ to be smooth and bounded together with the derivatives uniformly in γ (for some Λ and consequently for any $\Lambda \supset \Lambda(\omega)$).

Definition 4 We define the covariant derivative $\nabla^\Gamma\omega$ of the form $\omega \in \mathcal{F}\Omega^n$ as the mapping

$$\Gamma_X \ni \gamma \mapsto \nabla^\Gamma\omega(\gamma) := (\nabla^X_x \omega(\gamma))_{x\in\gamma} \in T_\gamma\Gamma_X \otimes (\wedge^n(T_\gamma\Gamma_X))$$

if for all $\gamma \in \Gamma_X$ and $x \in \gamma$ the form $\omega_x(\gamma, \bullet)$ is differentiable at x and the $\nabla^\Gamma\omega(\gamma)$ just defined indeed belongs to $T_\gamma\Gamma_X \otimes (\wedge^n(T_\gamma\Gamma_X))$.

Remark 3 For each $\omega \in \mathcal{F}\otimes^n$, the covariant derivative $\nabla^\Gamma\omega$ exists, and moreover only a finite number of the coordinates $\nabla^\Gamma\omega(\gamma)_{x,[x_1,\dots,x_n]_d}$ in the decomposition

$$T_\gamma\Gamma_X \otimes \big(\wedge^n (T_\gamma\Gamma_X)\big) = \bigoplus_{x\in\gamma,\, [x_1,\dots,x_n]_d\subset\gamma} T_xX \otimes (T_{x_1}X \wedge \cdots \wedge T_{x_n}X)$$

are not equal to zero.

Proposition 1 *For arbitrary* $\omega^{(1)}, \omega^{(2)} \in \mathcal{F}\Omega^n$, *we have*

$$\begin{aligned}\nabla^\Gamma\langle\omega^{(1)}(\gamma), \omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} &= \\ &= \langle\nabla^\Gamma\omega^{(1)}(\gamma), \omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} + \langle\omega^{(1)}(\gamma), \nabla^\Gamma\omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}.\end{aligned}$$

Proof. We have, for any fixed $\gamma \in \Gamma_X$ and $x \in \gamma$,

$$\begin{aligned}\nabla^X_x\langle\omega^{(1)}(\gamma), \omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} &= \nabla^X_x\langle\omega^{(1)}_x(\gamma, x), \omega^{(2)}_x(\gamma, x)\rangle_{\wedge^n(T_\gamma\Gamma_X)} \\ = \langle\nabla^X_x\omega^{(1)}(\gamma), \omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} &+ \langle\omega^{(1)}(\gamma), \nabla^X_x\omega^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)},\end{aligned}$$

because of the usual properties of the covariant derivative ∇^X_x. ■

2.3 Square integrable forms

In this subsection, we will consider spaces of forms over the configuration space Γ_X which are square integrable with respect to a Poisson measure.

Let m be the volume measure on X, let $\rho\colon X \to \mathbb{R}$ be a measurable function such that $\rho > 0$ m-a.e., and $\rho^{1/2} \in H^{1,2}_{\mathrm{loc}}(X)$, and define the measure $\sigma(dx) := \rho(x)\, m(dx)$. Here, $H^{1,2}_{\mathrm{loc}}(X)$ denotes the local Sobolev space of order 1 in $L^2_{\mathrm{loc}}(X; m)$. Then, σ is a non-atomic Radon measure on X.

Let π_σ stand for the Poisson measure on Γ_X with intensity σ. This measure is characterized by its Fourier transform

$$\int_{\Gamma_X} e^{i\langle f,\gamma\rangle}\, \pi_\sigma(d\gamma) = \exp \int_X (e^{if(x)} - 1)\, \sigma(dx), \qquad f \in C_0(X).$$

Let $F\in L^1(\Gamma_X;\pi_\sigma)$ be cylindrical, that is, there exits a compact $\Lambda\subset X$ such that $F(\gamma)=F(\gamma_\Lambda)$. Then, one has the following formula, which we will use many times:

$$\int_{\Gamma_X} F(\gamma)\,\pi_\sigma(d\gamma)=e^{-\sigma(\Lambda)}\sum_{n=0}^\infty\frac{1}{n!}\int_{\Lambda^n}F(\{x_1,\dots,x_n\})\,\sigma(dx_1)\cdots\sigma(dx_n). \tag{9}$$

Since the measure σ is non-atomic, the sets $\{(x_1,\dots,x_n)\in\Lambda^n : x_i=x_j\}$, $i,j=1,\dots,n$, $i\neq j$, have zero $\sigma(dx_1)\cdots\sigma(dx_n)$ measure, and therefore the expression on the right hand side of (9) is well-defined.

We define on the set $\mathcal{F}\Omega^n$ the L^2-scalar product with respect to the Poisson measure:

$$(\omega^{(1)},\omega^{(2)})_{L^2_{\pi_\sigma}\Omega^n}:=\int_{\Gamma_X}\langle\omega^{(1)}(\gamma),\omega^{(2)}(\gamma)\rangle_{\wedge^n T_\gamma\Gamma_X}\,\pi_\sigma(d\gamma). \tag{10}$$

As easily seen, for each $\omega\in\mathcal{F}\Omega^n$, the function $\langle\omega(\gamma),\omega(\gamma)\rangle_{\wedge^n T_\gamma\Gamma_X}$ is polynomially bounded on Γ_X, and therefore it belongs to all $L^p(\Gamma_X;\pi_\sigma)$, $p\geq 1$. Moreover, $(\omega,\omega)_{L^2_{\pi_\sigma}\Omega^n}>0$ if ω is not identically zero. Hence, we can define the Hilbert space

$$L^2_{\pi_\sigma}\Omega^n:=L^2(\Gamma_X\to\wedge^n T\Gamma_X;\pi_\sigma)$$

as the closure of $\mathcal{F}\Omega^n$ in the norm generated by the scalar product (10).

From now on, we consider the case of 1-forms only and suppose that $\dim X\geq 2$. We give another description of the spaces $L^2_{\pi_\sigma}\Omega^1$. Let us recall the following well-known result (Mecke identity, see e.g. [29]):

$$\int_{\Gamma_X}\int_X f(\gamma,x)\,\gamma(dx)\,\pi(d\gamma)=\int_{\Gamma_X}\int_X f(\gamma\cup\{x\},x)\,\sigma(dx)\,\pi_\sigma(d\gamma) \tag{11}$$

for any measurable bounded $f:\Gamma_X\times X\to\mathbb{R}^1$.

Let us introduce the notations

$$L^2_\sigma\Omega^1(X):=L^2(X\to TX;\sigma),\qquad L^2_{\pi_\sigma}(\Gamma_X):=L^2(\Gamma_X\to\mathbb{R}^1;\pi_\sigma).$$

Proposition 2 *The space $L^2_{\pi_\sigma}\Omega^1$ is isomorphic to the space $L^2_{\pi_\sigma}(\Gamma_X)\otimes L^2_\sigma\Omega^1(X)$ with the isomorphism I^1 given by the formula*

$$(I^1V)(\gamma,x):=V(\gamma\cup\{x\})_x,\qquad \gamma\in\Gamma_X,\ x\in X. \tag{12}$$

Proof. Let us specify the scalar product of two cylinder 1-forms $V,W\in\mathcal{F}\Omega^1$. We have:

$$(W,V)_{L^2_{\pi_\sigma}\Omega^1}=\int_{\Gamma_X}\langle W(\gamma),V(\gamma)\rangle_\gamma\,\pi_\sigma(d\gamma)$$

$$= \int_{\Gamma_X} \int_X \langle W(\gamma)_x, V(\gamma)_x \rangle_\gamma \; \gamma(dx)\, \pi_\sigma(d\gamma)$$
$$= \int_{\Gamma_X} \int_X \langle W(\gamma \cup \{x\})_x, V(\gamma \cup \{x\})_x \rangle_\gamma \; \gamma(dx)\, \pi_\sigma(d\gamma),$$

because $\gamma \cup \{x\} = \gamma$ for $x \in \gamma$. The application of the Mecke identity to the function

$$f(\gamma, x) = \langle V(\gamma \cup \{x\})_x, W(\gamma \cup \{x\})_x \rangle_\gamma$$

shows that

$$(W, V)_{L^2_{\pi_\sigma}\Omega^1} = \int_{\Gamma_X} \int_X \langle V(\gamma \cup \{x\})_x, W(\gamma \cup \{x\})_x \rangle_\gamma \; \sigma(dx)\, \pi_\sigma(d\gamma).$$

The space $\mathcal{F}\Omega^1$ is, by definition, dense in $L^2_{\pi_\sigma}\Omega^1$, and so it remains only to show that $I^1(\mathcal{F}\Omega^1)$ is dense in $L^2_{\pi_\sigma}(\Gamma_X) \otimes L^2_\sigma\Omega^1(X)$.

For $F \in \mathcal{F}C^\infty_{\mathrm{b}}(\Gamma_X)$, and $\nu \in \Omega^1_0(X)$ (the set of smooth 1-forms on X with compact support), we define a form $V(\gamma) = (V(\gamma)_x)_{x\in\gamma}$ by setting

$$V(\gamma)_x := F(\gamma \setminus \{x\})\nu(x). \tag{13}$$

Evidently, we have $V \in \mathcal{F}\Omega^1$, and

$$\left(I^1 V\right)(\gamma, x) = F(\gamma)\nu(x) \tag{14}$$

for each γ and any $x \notin \gamma$. Since each $\gamma \in \Gamma_X$ is a subset of X of zero m measure, we conclude from (14) that

$$I^1 V = F \otimes \nu. \tag{15}$$

Noting that the linear span of such $F \otimes \nu$ is dense in $L^2_{\pi_\sigma}(\Gamma_X) \otimes L^2_\sigma\Omega^1(X)$, we obtain the result. ■

In what follows, we will denote by $\mathcal{D}\Omega^1$ the linear span of forms V defined by (13). As we already noticed in the proof of Proposition 2, $\mathcal{D}\Omega^1 \subset \mathcal{F}\Omega^1$ and is dense in $L^2_{\pi_\sigma}\Omega^1$.

Corollary 1 *We have the unitary isomorphism*

$$\mathcal{I} : L^2_{\pi_\sigma}\Omega^1 \to \mathrm{Exp}\left(L^2(X;\sigma)\right) \otimes L^2_\sigma\Omega^1(X)$$

given by

$$\mathcal{I} = (U \otimes \mathbf{1})\, I^1,$$

where U is the unitary isomorphism between the Poisson space $L^2_{\pi_\sigma}(\Gamma_X)$ and the symmetric Fock space $\mathrm{Exp}\,(L^2(X;\sigma))$*, see e.g.* [7].

3 Dirichlet operators on differential forms over configuration spaces

In this section, we introduce Dirichlet operators associated with the Poisson measure on Γ_X which act in the space $L^2_{\pi_\sigma}\Omega^1$. These operators generalize the notions of Bochner and de Rham–Witten Laplacians on finite dimensional manifolds.

In the two first subsections, we recall some known facts and definitions concerning Dirichlet operators of Poisson measures on configuration spaces and Laplace operators on differential forms over finite-dimensional manifolds.

3.1 The intrinsic Dirichlet operator on functions

In this subsection, we recall some theorems from [7] which concern the intrinsic Dirichlet operator in the space $L^2_{\pi_\sigma}(\Gamma_X)$, to be used later.

Let us recall that the logarithmic derivative of the measure σ is given by the vector field

$$X \ni x \mapsto \beta_\sigma(x) := \frac{\nabla^X \rho(x)}{\rho(x)} \in T_x X$$

(where as usual $\beta_\sigma := 0$ on $\{\rho = 0\}$). We wish now to define the notion of logarithmic derivative of the Poisson measure, and for this we need a generalization of the notion of vector field.

For each $\gamma \in \Gamma_X$, consider the triple

$$T_{\gamma,\infty}\Gamma_X \supset T_\gamma \Gamma_X \supset T_{\gamma,0}\Gamma_X.$$

Here, $T_{\gamma,0}\Gamma_X$ consists of all finite sequences from $T_\gamma\Gamma_X$, and $T_{\gamma,\infty}\Gamma_X := (T_{\gamma,0}\Gamma_X)'$ is the dual space, which consists of all sequences $V(\gamma) = (V(\gamma)_x)_{x\in\gamma}$, where $V(\gamma)_x \in T_x X$. The pairing between any $V(\gamma) \in T_{\gamma,\infty}\Gamma_X$ and $v(\gamma) \in T_{\gamma,0}\Gamma_X$ with respect to the zero space $T_\gamma\Gamma_x$ is given by

$$\langle V(\gamma), v(\gamma)\rangle_\gamma = \sum_{x\in\gamma} \langle V(\gamma)_x, v(\gamma)_x\rangle_x$$

(the series is, in fact, finite). From now on, under a vector field over Γ_X we will understand mappings of the form $\Gamma_X \ni \gamma \mapsto V(\gamma) \in T_{\gamma,\infty}\Gamma_X$.

The logarithmic derivative of the Poisson measure π_σ is defined as the vector field

$$\Gamma_X \ni \gamma \mapsto B_{\pi_\sigma}(\gamma) = (\beta_\sigma(x))_{x\in\gamma} \in T_{\gamma,\infty}\Gamma_X \tag{16}$$

(i.e., the logarithmic derivative of the Poisson measure is the lifting of the logarithmic derivative of the underlying measure).

The following theorem is a version of Theorem 3.1 in [7] (for more general classes of functions and vector fields).

Theorem 1 (Integration by parts formula on the Poisson space) *For arbitrary $F^{(1)}, F^{(2)} \in \mathcal{F}C_{\mathrm{b}}^{\infty}(\Gamma_X)$ and a smooth cylinder vector field $V \in \mathcal{FV}(\Gamma_X)$ $(:= \mathcal{F}\Omega^1)$, we have*

$$\int_{\Gamma_X} \nabla_V^{\Gamma} F^{(1)}(\gamma) F^{(2)}(\gamma)\, \pi_\sigma(d\gamma) = -\int_{\Gamma_X} F^{(1)}(\gamma) \nabla_V^{\Gamma} F^{(2)}(\gamma)\, \pi_\sigma(d\gamma)$$
$$-\int_{\Gamma_X} F^{(1)}(\gamma) F^{(2)}(\gamma) \big[\, \langle B_{\pi_\sigma}(\gamma), V(\gamma)\rangle_\gamma + \operatorname{div}^{\Gamma} V(\gamma)\big]\, \pi_\sigma(d\gamma),$$

where the divergence $\operatorname{div}^{\Gamma} V(\gamma)$ of the vector field V is given by

$$\operatorname{div} V(\gamma) = \sum_{x\in\gamma} \operatorname{div}_x^X V(\gamma) = \langle \operatorname{div}_\bullet^X V(\gamma), \gamma\rangle,$$
$$\operatorname{div}_x^X V(\gamma) := \operatorname{div}^X V_x(\gamma, x), \qquad x \in \gamma,$$

div^X denoting the divergence on X with respect to the volume measure m.

Proof. The theorem follows from formula (9) and the usual integration by parts formula on the space $L^2(\Lambda^n, \sigma^{\otimes n})$ (see also the proof of Theorem 3 below). ■

Following [7], we consider the intrinsic pre-Dirichlet form on the Poisson space

$$\mathcal{E}_{\pi_\sigma}(F^{(1)}, F^{(2)}) = \int_{\Gamma_X} \langle \nabla^{\Gamma} F^{(1)}(\gamma), \nabla^{\Gamma} F^{(2)}(\gamma)\rangle_\gamma\, \pi_\sigma(d\gamma) \tag{17}$$

with domain $D(\mathcal{E}_{\pi_\sigma}) := \mathcal{F}C_{\mathrm{b}}^{\infty}(\Gamma_X)$. By using the fact that the measure π_σ has all moments finite, one can show that the expression (17) is well-defined.

Let H_σ denote the Dirichlet operator in the space $L^2(X;\sigma)$ associated to the pre-Dirichlet form

$$\mathcal{E}_\sigma(\varphi, \psi) = \int_X \langle \nabla^X \varphi(x), \nabla^X \psi(x)\rangle_x\, \sigma(dx), \qquad \varphi, \psi \in \mathcal{D}.$$

This operator acts as follows:

$$H_\sigma \varphi(x) = -\Delta^X \varphi(x) - \langle \beta_\sigma(x), \nabla^X \varphi(x)\rangle_x, \qquad \varphi \in \mathcal{D},$$

where $\Delta^X := \operatorname{div}^X \nabla^X$ is the Laplace–Beltrami operator on X.

Then, by using Theorem 1, one gets

$$\mathcal{E}_{\pi_\sigma}(F^{(1)}, F^{(2)}) = \int_{\Gamma_X} H_{\pi_\sigma} F^{(1)}(\gamma) F^{(2)}(\gamma)\, \pi_\sigma(d\gamma), \qquad F^{(1)}, F^{(2)} \in \mathcal{F}C_{\mathrm{b}}^{\infty}(\Gamma_X). \tag{18}$$

Here, the intrinsic Dirichlet operator H_{π_σ} is given by

$$H_{\pi_\sigma} F(\gamma) : = \sum_{x\in\gamma} H_{\sigma,x} F(\gamma) \equiv \langle H_{\sigma,\bullet} F(\gamma), \gamma\rangle,$$

$$H_{\sigma,x}F(\gamma) := H_\sigma F_x(\gamma, x), \qquad x \in \gamma, \tag{19}$$

so that the operator H_{π_σ} is the lifting to $L^1(\Gamma_X; \pi_\sigma)$ of the operator H_σ in $L^2(X; \sigma)$.

Upon (18), the pre-Dirichlet form $\mathcal{E}_{\pi_\sigma}$ is closable, and we preserve the notation for the closure of this form.

Theorem 2 [7] *Suppose that $(H_\sigma, \mathcal{D})$ is essentially self-adjoint on $L^2(X; \sigma)$. Then, the operator H_{π_σ} is essentially self-adjoint on $\mathcal{FC}_{\mathrm{b}}^\infty(\Gamma_X)$.*

Remark 4 This theorem was proved in [7], Theorem 5.3. (We have already mentioned in Remark 1 that the inclusion $\mathcal{FC}_{\mathrm{b}}^\infty(\mathcal{D}, \Gamma_X) \subset \mathcal{FC}_{\mathrm{b}}^\infty(\Gamma_X)$ holds.) We would like to stress that this result is based on the theorem which says that the image of the operator H_{π_σ} under the isomorphism U between the Poisson space and the Fock space $\operatorname{Exp}(L^2(X; \sigma))$over $L^2(X; \sigma)$ is the differential second quantization $d \operatorname{Exp} H_\sigma$ of the operator H_σ.

Remark 5 In what follows, we will always assume that the conditions of the theorem are satisfied. It is true e.g. in the case where $\|\beta_\sigma\|_{TX} \in L^p_{\mathrm{loc}}(X; \sigma)$ for some $p > \dim X$, see [7].

Finally, we mention the important fact that the diffusion process which is properly associated with the Dirichlet form $(\mathcal{E}_{\pi_\sigma}, D(\mathcal{E}_{\pi_\sigma}))$ is the usual independent infinite particle process (or distorted Brownian motion), cf. [7].

3.2 Laplacians on differential forms over finite-dimensional manifolds

We recall now some facts on the Bochner and de Rham–Witten Laplacians on differential forms over a finite-dimensional manifold.

Let M be a Riemannian manifold equipped with the measure $\mu(dx) = e^{\phi(x)}dx$, dx being the volume measure and ϕ a C_{b}^2-function on M. We consider a Hilbert bundle

$$\mathcal{H}_x \mapsto x \in M$$

over M equipped with a smooth connection, and denote by ∇ the corresponding covariant derivative in the spaces of sections of this bundle. Let $L^2(M \to \mathcal{H}; \mu)$ be the space of μ-square integrable sections. The operator

$$H_\mu^B := \nabla_\mu^* \nabla$$

in $L^2(M \to \mathcal{H}; \mu)$, where ∇_μ^* is the adjoint of ∇, will be called the Bochner Laplacian associated with the measure μ. Differentiability of μ implies that $\nabla_\mu^* \nabla$ is a uniquely defined self-adjoint operator. One can easily write the corresponding differential

expression on the space of twice differentiable sections. In the case where $\phi \equiv 0$ and $\mathcal{H}_x = \wedge^n(T_xM)$, we obtain the classical Bochner Laplacian on differential forms (see [19]).

Now, let d be the exterior differential in spaces of differential forms over M. The operator

$$H_\mu^R := d_\mu^* d + dd_\mu^*$$

acting in the space of μ-square integrable forms, where d_μ^* is the adjoint of d, will be called the de Rham Laplacian associated with the measure μ (or the Witten Laplacian associated with ϕ, see [19]).

The relation of the Bochner and de Rham–Witten Laplacians on differential forms is given by the Weitzenböck formula, which in the case of 1-forms has the following form (see[19], [22]):

$$H_\mu^R u(x) = H_\mu^B u(x) + R_\mu(x)u(x),$$

where

$$R_\mu(x) := R(x) - \nabla^X \beta_\mu(x). \tag{20}$$

Here, $R(x) \in \mathcal{L}(T_xM)$ is the usual Weitzenböck correction term:

$$R(x) := \sum_{i,j=1}^{\dim M} \mathrm{Ric}_{ij}(x) a_i^* a_j,$$

where Ric is the Ricci tensor on X, and a_i^* and a_j are the creation and annihilation operators, respectively.

3.3 Bochner Laplacian on 1-forms over the Poisson space

Let us consider the pre-Dirichlet form

$$\mathcal{E}_{\pi_\sigma}^B(W^{(1)}, W^{(2)}) = \int_{\Gamma_X} \langle \nabla^\Gamma W^{(1)}(\gamma), \nabla^\Gamma W^{(2)}(\gamma) \rangle_{T_\gamma\Gamma_X \otimes T_\gamma\Gamma_X}\, \pi_\sigma(d\gamma), \tag{21}$$

where $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^1$. Again using the fact that π_σ has finite moments, one shows that the function under the sign of integral in (21) is integrable with respect to π_σ.

Theorem 3 *For any $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^1$, we have*

$$\mathcal{E}_{\pi_\sigma}^B(W^{(1)}, W^{(2)}) = \int_{\Gamma_X} \left\langle H_{\pi_\sigma}^B W^{(1)}(\gamma), W^{(2)}(\gamma) \right\rangle_{T_\gamma\Gamma_X}\, \pi_\sigma(d\gamma),$$

where $H^B_{\pi_\sigma}$ is the operator in the space $L^2_{\pi_\sigma}\Omega^1$ given by

$$H^B_{\pi_\sigma} W = -\Delta^\Gamma W - \left\langle \nabla^\Gamma W, B_{\pi_\sigma}(\gamma) \right\rangle_\gamma, \qquad W \in \mathcal{F}\Omega^1. \tag{22}$$

Here,

$$\Delta^\Gamma W(\gamma) := \sum_{x\in\gamma} \Delta^X_x W(\gamma) \equiv \left\langle \Delta^\Gamma_\bullet W(\gamma), \gamma \right\rangle, \tag{23}$$

where Δ^X_x is the Bochner Laplacian of the bundle $T_{\gamma_y}\Gamma_X \mapsto y \in \mathcal{O}_{\gamma,x}$ with the volume measure.

Proof. Let us fix $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^1$. Let Λ be an open bounded set in X such that $\Lambda(W^{(1)}) \subset \Lambda$, $\Lambda(W^{(2)}) \subset \Lambda$. Then, by using (9),

$$\begin{aligned}
&\int_{\Gamma_X} \langle \nabla^\Gamma W^{(1)}(\gamma), \nabla^\Gamma W^{(2)} \rangle_{T_\gamma\Gamma_X \otimes T_\gamma\Gamma_X}\, \pi_\sigma(d\gamma) = \\
&= e^{-\sigma(\Lambda)} \sum_{k=0}^\infty \frac{1}{k!} \int_{\Lambda^k} \sum_{i=1}^k \langle \nabla^X_{x_i} W^{(1)}(\{x_1,\dots,x_k\}), \\
&\nabla^X_{x_i} W^{(2)}(\{x_1,\dots,x_k\}) \rangle_{T_{x_i}X \otimes T_{\{x_1,\dots,x_k\}}\Gamma_X}\, \sigma(dx_1)\cdots\sigma(dx_k) \\
&= e^{-\sigma(\Lambda)} \sum_{k=0}^\infty \frac{1}{k!} \sum_{i=1}^k \int_{\Lambda^k} \langle \Delta^X_{x_i} W^{(1)}(\{x_1,\dots,x_k\}) \\
&+ \langle \nabla^X_{x_i} W^{(1)}(\{x_1,\dots,x_k\}), \beta_\sigma(x_i) \rangle_{T_{x_i}X}, \\
&W^{(2)}(\{x_1,\dots,x_k\}) \rangle_{T_{\{x_1,\dots,x_k\}}\Gamma_X}\, \sigma(dx_1)\cdots\sigma(dx_k) \\
&= \int_{\Gamma_X} \langle H^B_{\pi_\sigma} W^{(1)}(\gamma), W^{(2)}(\gamma) \rangle_{T_\gamma\Gamma_X}\, \pi_\sigma(d\gamma). \quad \blacksquare
\end{aligned}$$

Remark 6 We can rewrite the action of the operator $H^B_{\pi_\sigma}$ in the two following forms:

1) We have from (22) and (23) that

$$H^B_{\pi_\sigma} W(\gamma) = \sum_{x\in\gamma} H^B_{\sigma,x} W(\gamma) \equiv \left\langle H^B_{\sigma,\bullet} W(\gamma), \gamma \right\rangle, \qquad W(\gamma) \in \mathcal{F}\Omega^1, \tag{24}$$

where

$$H^B_{\sigma,x} W(\gamma) := -\Delta^X_x W(\gamma) - \left\langle \nabla^X_x W(\gamma), \beta_\sigma(x) \right\rangle_x. \tag{25}$$

Thus, the operator $H^B_{\sigma,x}$ is the lifting of the Bochner Laplacian on X with the measure σ.

2) As easily seen, the operator $H^B_{\pi_\sigma}$ preserves the space $\mathcal{F}\Omega^1$, and we can always take $\Lambda(H^B_{\pi_\sigma}W)=\Lambda(W)$. Then for any open bounded $\Lambda\supset\Lambda(W)$

$$(H^B_{\pi_\sigma}W)_{\Lambda,\gamma}=H^B_{\sigma,\Lambda\cap\gamma}W_{\Lambda,\gamma}, \tag{26}$$

where $H^B_{\sigma,\Lambda\cap\gamma}$ is the Bochner Laplacian of the manifold $X^{\Lambda\cap\gamma}:=\times_{x\in\Lambda\cap\gamma}X_x$, $X_x\equiv X$, with the product measure $\sigma^{\Lambda\cap\gamma}:=\otimes_{x\in\Lambda\cap\gamma}\sigma_x$, $\sigma_x\equiv\sigma$ (cf. (8)).

It follows from Theorem 3 that the pre-Dirichlet form $\mathcal{E}^B_{\pi_\sigma}$ is closable in the space $L^2_{\pi_\sigma}\Omega^1$. The generator of its closure (being actually the Friedrichs extension of the operator $H^B_{\pi_\sigma}$, for which we will use the same notation) will be called the Bochner Laplacian on 1-forms over Γ_X corresponding to the Poisson measure π_σ.

For operators A and B acting in Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, respectively, we introduce the operator $A\boxplus B$ in $\mathcal{H}\otimes\mathcal{K}$ by

$$A\boxplus B:=A\otimes\mathbf{1}+\mathbf{1}\otimes B.$$

Proposition 3 1) *On $\mathcal{D}\Omega^1$ we have*

$$I^1\,H^B_{\pi_\sigma}=\left(H_{\pi_\sigma}\boxplus H^B_\sigma\right)\,I^1. \tag{27}$$

2) *$\mathcal{D}\Omega^1$ is a domain of essential self-adjointness of $H^B_{\pi_\sigma}$.*

Proof. 1) Let $W\in\mathcal{D}\Omega^1$. Then, for some $F\in\mathcal{F}C^\infty_{\mathrm{b}}(\Gamma_X)$, $\omega\in\Omega^1_0(X)$, and any $\gamma\in\Gamma_X$, $x,z\in\gamma$, $y\in\mathcal{O}_x$, we have

$$W_x(\gamma,y)_z=\begin{cases}F((\gamma\setminus\{x,z\})\cup\{y\})\omega(z), & z\neq y,\\ F(\gamma\setminus\{x\})\omega(y), & z=y\end{cases}$$

Thus

$$H^B_{\sigma,x}W(\gamma)_z=\begin{cases}H_{\sigma,x}F(\gamma\setminus\{z\})\omega(z), & z\neq x,\\ F(\gamma\setminus\{z\})H^B_\sigma\omega(z), & z=x.\end{cases}$$

Formula (27) follows now from (24) and (14).

2) The statement follows from (27) and the essential self-adjointness of H_{π_σ} on $\mathcal{F}C^\infty_{\mathrm{b}}(\Gamma_X)$ (Theorem 2) and H^B_σ on $\Omega^1_0(X)$ (the latter fact can be shown by standard methods similar to [21], [22]) by the theory of operators admitting separation of variables [16, Ch.6]. ■

We give also a Fock space representation of the operator $H^B_{\pi_\sigma}$. Corollary 1 implies the following

Corollary 2 *We have*

$$\mathcal{I}H^B_{\pi_\sigma}\mathcal{I}^{-1}=d\,\mathrm{Exp}\,H_\sigma\boxplus H^B_\sigma,$$

cf. Remark 4.

3.4 De Rham Laplacian on 1-forms over the Poisson space

We define the linear operator

$$d^\Gamma : \mathcal{F}\Omega^1 \to \mathcal{F}\Omega^2$$

by

$$(d^\Gamma W)(\gamma) := \sqrt{2}\ \mathrm{AS}(\nabla_x^X W(\gamma)), \tag{28}$$

where $\mathrm{AS} : (T_\gamma \Gamma_X)^{\otimes 2} \to (T_\gamma \Gamma_X)^{\wedge 2}$ is the antisymmetrization operator. It follows from this definition that

$$(d^\Gamma W)(\gamma) = \sum_{x\in\gamma} (d_x^X W)(\gamma), \tag{29}$$

where

$$\begin{aligned}(d_x^X W)(\gamma) :&= \sum_{y\in\gamma} d^X (W_x(\gamma, x)_y) \\ &= \sum_{y\in\gamma} \sqrt{2}\ \mathrm{AS}(\nabla^X W_x(\gamma, x)_y)\end{aligned} \tag{30}$$

with $\mathrm{AS} : T_x X \otimes T_y X \to T_x X \wedge T_y X$ being again the antisymmetrization. This implies that we have indeed the inclusion $d^\Gamma W \in \mathcal{F}\Omega^2$ for each $W \in \mathcal{F}\Omega^1$.

Suppose that, for $W \in \mathcal{F}\Omega^1$, $\gamma \in \gamma_X$, and $x, y \in \gamma$, the 1-form $W_x(\gamma, \bullet)_y$ has, in local coordinates on the manifold X, the following form:

$$W_x(\gamma, \bullet)_y = u(\bullet) h, \qquad u\colon \mathcal{O}_{\gamma,x} \to \mathbb{R},\ h \in T_y. \tag{31}$$

Then, we have

$$\mathrm{AS}(\nabla^X W_x(\gamma, x)_y) = \nabla^X u(x) \wedge h, \tag{32}$$

which, upon (30), describes the action of d_x^X.

Let us consider d^Γ as an operator acting from the space $L^2_{\pi_\sigma}\Omega^1$ into $L^2_{\pi_\sigma}\Omega^2$. Analogously to the proof of Theorem 3, we get the following formula for the adjoint operator $(d^\Gamma_{\pi_\sigma})^*$ restricted to $\mathcal{F}\Omega^2$:

$$(d^\Gamma_{\pi_\sigma})^* W(\gamma) = \sum_{x\in\gamma} (d^X_{\sigma,x})^* W(\gamma), \qquad W \in \mathcal{F}\Omega^2, \tag{33}$$

where

$$(d^X_{\sigma,x})^* W(\gamma) = \sum_{y\in\gamma} (d^X_{\sigma,x})^* W_x(\gamma, x)_{[x,y]}. \tag{34}$$

Suppose that, in local coordinates on the manifold X, the form $W_x(\bullet, \bullet)_{[x,y]}$ has the representation

$$W_x(\gamma, \bullet)_{[x,y]} = w(\bullet) h_1 \wedge h_2, \qquad w\colon \mathcal{O}_{\gamma,x} \to \mathbb{R},\ h_1 \in T_x X,\ h_2 \in T_y X. \tag{35}$$

Then, taking to notice (32), one concludes that

$$(d^X_{\sigma,x})^*(W_x(\gamma,x)_{[x,y]}) = -\frac{1}{\sqrt{2}}\left[\left(\langle\nabla^X w(x),h_1\rangle_x + w(x)\langle\beta_\sigma(x),h_1\rangle_x\right)h_2\right.$$
$$\left. - \delta_{x,y}\left(\langle\nabla^X w(x),h_2\rangle_x + w(x)\langle\beta_\sigma(x),h_2\rangle_x\right)h_1\right]. \quad (36)$$

Here,

$$\delta_{x,y} = \begin{cases} 1, & \text{if } x=y, \\ 0, & \text{otherwise.} \end{cases}$$

In what follows, we will suppose for simplicity that the function ρ is infinite differentiable on X and $\rho(x)>0$ for all $x\in X$. Then, by (33)–(36)

$$(d^\Gamma_{\pi_\sigma})^* : \mathcal{F}\Omega^2 \to \mathcal{F}\Omega^1.$$

We set also

$$d^\Gamma : \mathcal{FC}^\infty_{\mathrm{b}}(\Gamma_X) \to \mathcal{F}\Omega^1, \qquad d^\Gamma := \nabla^\Gamma. \quad (37)$$

Evidently, the restriction to $\mathcal{F}\Omega^1$ of the adjoint of d^Γ considered as an operator acting from $L^2_{\pi_\sigma}(\Gamma_X)$ into $L^2_{\pi_\sigma}\Omega^1$ is given by

$$(d^\Gamma_{\pi_\sigma})^* : \mathcal{F}\Omega^1 \to \mathcal{FC}^\infty_{\mathrm{b}}(\Gamma_X), \qquad (d^\Gamma_{\pi_\sigma})^* V(\gamma) = -\operatorname{div}^\Gamma V(\gamma) - \langle V(\gamma), B_{\pi_\sigma}(\gamma)\rangle_\gamma. \quad (38)$$

For $n\in\mathbb{N}$, we define the pre-Dirichlet form $\mathcal{E}^R_{\pi_\sigma}$ by

$$\mathcal{E}^R_{\pi_\sigma}(W^{(1)},W^{(2)}) := \int_{\Gamma_X}\left[\langle d^\Gamma W^{(1)}(\gamma), d^\Gamma W^{(2)}(\gamma)\rangle_{\wedge^2(T_\gamma\Gamma_X)}\right.$$
$$\left. + \langle (d^\Gamma_{\pi_\sigma})^* W^{(1)}(\gamma), (d^\Gamma_{\pi_\sigma})^* W^{(2)}(\gamma)\rangle_{T_\gamma\Gamma_X}\right]\pi_\sigma(d\gamma),$$

where $W^{(1)},W^{(2)}\in D(\mathcal{E}^R_{\pi_\sigma}) := \mathcal{F}\Omega^1$.

The next theorem follows easily from (28)–(38).

Theorem 4 *For any $W^{(1)},W^{(2)}\in\mathcal{F}\Omega^1$, we have*

$$\mathcal{E}^R_{\pi_\sigma}(W^{(1)},W^{(2)}) = \int_{\Gamma_X}\langle H^R_{\pi_\sigma} W(\gamma)^{(1)}(\gamma), W(\gamma)^{(2)}\rangle_\gamma\,\pi_\sigma(d\gamma).$$

Here,

$$H^R_{\pi_\sigma} := d^\Gamma(d^\Gamma_{\pi_\sigma})^* + (d^\Gamma_{\pi_\sigma})^* d^\Gamma, \qquad D(H^R_{\pi_\sigma}) := \mathcal{F}\Omega^1,$$

is an operator in the space $L^2_{\pi_\sigma}\Omega^1$. It can be represented as follows:

$$H^R_{\pi_\sigma} W(\gamma) = \sum_{x\in\gamma} H^R_{\sigma,x} W(\gamma) \equiv \left\langle H^R_{\sigma,\bullet} W(\gamma),\gamma\right\rangle, \quad (39)$$

where

$$H^R_{\sigma,x} = d^X_x(d^X_{\sigma,x})^* + (d^X_{\sigma,x})^* d^X_x. \quad (40)$$

From Theorem 4 we conclude that the pre-Dirichlet form $\mathcal{E}^R_{\pi_\sigma}$ is closable in the space $L^2_{\pi_\sigma}\Omega^1$.

The generator of its closure (being actually the Friedrichs extension of the operator $H^R_{\pi_\sigma}$, for which we will use the same notation) will be called the de Rham Laplacian on Γ_X corresponding to the Poisson measure π_σ. By (39) and (40), $H^R_{\pi_\sigma}$ is the lifting of the de Rham Laplacian on X with measure σ.

Remark 7 Similarly to (26), the operator $H^B_{\pi_\sigma}$ preserves the space $\mathcal{F}\Omega^1$, and we can always take $\Lambda(H^B_{\pi_\sigma}W) = \Lambda(W)$. Then for any open bounded $\Lambda \supset \Lambda(W)$, we have

$$(H^R_{\pi_\sigma}W)_{\Lambda,\gamma} = H^R_{\sigma,\Lambda\cap\gamma}W_{\Lambda,\gamma}, \tag{41}$$

where $H^R_{\sigma,\Lambda\cap\gamma}$ is the de Rham Laplacian of the manifold $X^{\Lambda\cap\gamma}$ with the product measure $\sigma^{\Lambda\cap\gamma}$.

Proposition 4 1) *On $\mathcal{D}\Omega^1$ we have*

$$I^1\, H^R_{\pi_\sigma} = \left(H_{\pi_\sigma} \boxplus H^R_\sigma\right)\, I^1. \tag{42}$$

2) *$\mathcal{D}\Omega^1$ is a domain of essential self-adjointness of $H^R_{\pi_\sigma}$.*

Proof. 1) The proof is similar to that of (27). It is only necessary to note that, for a "constant" 1-form W such that $W(\gamma)_x = \nu(x)$, we have evidently $\left(H^R_{\sigma,x}w(\gamma)\right)_x = H^R_\sigma\omega(x)$.

2) The proof is similar to that of the corresponding statement for the Bochner Laplacian $H^B_{\pi_\sigma}$. ■

Remark 8 By similar methods, one can define Bochner and de Rham Laplacians on n-forms over Γ_X. An extension to this case of formulas (27) and (42) will have, however, a more complicated form.

3.5 Weitzenböck formula on the Poisson space

In this section, we will derive a generalization of the Weitzenböck formula to the case of the Poisson measure on the configuration space. In other words, we will derive a formula which gives a relation between the Bochner and de Rham Laplacians. We assume that the Weitzenböck correction term $R_\sigma(x) \in \mathcal{L}(T_xX)$ (cf. (20)) is bounded uniformly in $x \in X$.

Given an operator field

$$X \ni x \mapsto J(x) \in \mathcal{L}(T_xX) \tag{43}$$

on X (with $J(x)$ bounded uniformly in $x \in X$), we define the "diagonal" operator field

$$\Gamma_X \ni \gamma \mapsto \mathbf{J}(\gamma) \in \mathcal{L}(T_\gamma\Gamma_X), \tag{44}$$

using the decomposition (1). Thus, we can define the operator field $\mathbf{R}_\sigma(\gamma)$.

Theorem 5 (Weitzenböck formula on the Poisson space) *We have, for each* $W \in \mathcal{F}\Omega^1$,

$$H^R_{\pi_\sigma} W(\gamma) = H^B_{\pi_\sigma} W(\gamma) + \mathbf{R}_\sigma(\gamma) W(\gamma). \tag{45}$$

Proof. Let us fix $W \in \mathcal{F}\Omega^1$ and $\gamma \in \Gamma_X$. Let $\Lambda \subset X$ be an open bounded set such that $\Lambda \supset \Lambda(W)$ (cf. Definition 3), and let $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k}$ and $W_{\Lambda,\gamma}$ be as in Remark 2. We have then, according to (26) and (41),

$$\begin{aligned}(H^B_{\pi_\sigma} W)_{\Lambda,\gamma} &= H^B_{\sigma,\Lambda\cap\gamma} W_{\Lambda,\gamma},\\ (H^R_{\pi_\sigma} W)_{\Lambda,\gamma} &= H^R_{\sigma,\Lambda\cap\gamma} W_{\Lambda,\gamma},\end{aligned}$$

and the Weitzenböck formula for the manifold $X^{\Lambda\cap\gamma}$ and the measure $\sigma^{\Lambda\cap\gamma}$ implies that

$$H^R_{\sigma,\Lambda\cap\gamma} W_{\Lambda,\gamma}(y_1,\dots,y_k) = H^B_{\sigma,\Lambda\cap\gamma} W_{\Lambda,\gamma}(y_1,\dots,y_k) + R(y_1,\dots,y_k) W_{\Lambda,\gamma}(y_1,\dots,y_k),$$

where the correction term $R(y_1,\dots,y_k) \in \mathcal{L}(T_{(y_1,\dots,y_k)} X^{\Lambda\cap\gamma})$ is equal to the restriction of $\mathbf{R}_\sigma(\gamma)$ to the space $T_{(y_1,\dots,y_k)} X^{\Lambda\cap\gamma}$ (considered as a subspace of $T_\gamma \Gamma_X$), which is well-defined because of the "diagonal" character of $\mathbf{R}_\sigma(\gamma)$. It is now enough to remark that the forms $H^R_{\pi_\sigma} W$ and $H^B_{\pi_\sigma} W$ are completely defined by the corresponding forms $(H^R_{\pi_\sigma} W)_{\Lambda,\gamma}$ and $(H^B_{\pi_\sigma} W)_{\Lambda,\gamma}$, respectively ■

We can give also an intrinsic description of the correction term $\mathbf{R}^n_\sigma(\gamma)$. To this end, for each fixed $\gamma \in \Gamma_X$, we define the operator $R(\gamma)\colon T_{\gamma,0}\Gamma_X \to T_{\gamma,0}\Gamma_X$ as follows:

$$R(\gamma) := \sum_{x\in\gamma} R(\gamma, x),$$

$$R(\gamma,x)(V(\gamma)_y) := \delta_{x,y} \sum_{i,j=1}^d \mathrm{Ric}_{ij}(x) e_i \langle V(\gamma)_x, e_j\rangle_x, \qquad V(\gamma) \in T_{\gamma,0}\Gamma_X. \tag{46}$$

Here, $\{e_j\}_{j=1}^d$ is again a fixed orthonormal basis in the space $T_x X$ considered as a subspace of $T_\gamma \Gamma_X$.

Next, we note that

$$\begin{aligned}\nabla^\Gamma B_{\pi_\sigma}(\gamma) &= (\nabla^X_x B_{\pi_\sigma}(\gamma))_{x\in\gamma} = (\nabla^X_x (B_{\pi_\sigma}(\gamma)_y))_{x,y\in\gamma}\\ &= (\delta_{x,y} \nabla^X \beta_\sigma(y))_{x,y\in\gamma} \in (T_{\gamma,\infty}\Gamma_X)^{\otimes 2}.\end{aligned}$$

Hence, for any $V(\gamma) \in T_{\gamma,0}\Gamma_X$,

$$\begin{aligned}\nabla^\Gamma_V B_{\pi_\sigma}(\gamma) :&= \langle \nabla^\Gamma B_{\pi_\sigma}(\gamma), V(\gamma)\rangle_\gamma\\ &= \left(\sum_{y\in\gamma} \delta_{x,y} \langle \nabla^X \beta_\sigma(y), V(\gamma)_y\rangle_y \right)_{x\in\gamma}\end{aligned}$$

$$= \big(\langle \nabla^X \beta_\sigma(x), V(\gamma)_x\rangle_x\big)_{x\in\gamma} \in T_{\gamma,0}\Gamma_X. \tag{47}$$

Thus, $\nabla^\Gamma B_{\pi_\sigma}(\gamma)$ determines the linear operator in $T_{\gamma,0}\Gamma_X$ given by

$$T_{\gamma,0}\Gamma_X \ni V(\gamma) \mapsto \nabla^\Gamma B_{\pi_\sigma}(\gamma)V(\gamma) := \nabla^\Gamma_V B_{\pi_\sigma}(\gamma) \in T_{\gamma,0}\Gamma_X.$$

Proposition 5 *We have*

$$\mathbf{R}_\sigma(\gamma)W(\gamma) = R(\gamma)W(\gamma) - \nabla^\Gamma B_{\pi_\sigma}(\gamma)W(\gamma).$$

Proof. The proposition is derived from the definition of $\mathbf{R}_\sigma$ and formulas (47) and (46). ■

4 Probabilistic representations of the Bochner and de Rham Laplacians

Let $\xi_x(t)$ be the Brownian motion with the drift β_σ on X started at a point $x \in X$. We suppose the following:

- for each x, the process $\xi_x(t)$ has an infinite life-time;
- the semigroup
$$T_0(t)f(x) := \mathsf{E}\, f(\xi_x(t))$$
acting in the space of bounded measurable functions on X can be extended to a strongly continuous semigroup of contractions in $L^2(X;\sigma)$, and its generator H_0 is essentially self-adjoint on the space $\mathcal{D}$ (in this case $H_0 = -H_\sigma$).

It follows from the general theory of stochastic differential equations that these assumptions are satisfied if e.g. $\beta_\sigma \in C_b^4(X \to TX)$.

We denote by $\xi_\gamma(t)$ the corresponding independent particle process on Γ_X which starts at a point γ,

$$\xi_\gamma(t) = (\xi_x(t))_{x\in\gamma}.$$

Let

$$\mathbf{T}_0(t)F(\gamma) := \mathsf{E}\, F(\xi_\gamma(t)) \tag{48}$$

be the corresponding semigroup in the space of measurable bounded functions on Γ_X. It is shown in [7], that it can be extended to a strongly continuous semigroup in $L^2_{\pi_\sigma}(\Gamma_X)$ with the generator $\mathbf{H}_0 = -H_{\pi_\sigma}$ on $\mathcal{F}\mathcal{C}_{\mathrm{b}}^\infty(\Gamma_X)$.

Given the operator field (43), which is supposed to be continuous and symmetric (i.e., $J(x) = J(x)^*$), we define the operator

$$\mathbf{P}^J_{\xi_\gamma}(t) : T_{\xi_\gamma(t)}\Gamma_X \to T_\gamma\Gamma_X \tag{49}$$

by setting

$$(\mathbf{P}^J_{\xi_\gamma}(t)V)_x = (P^J_{\xi_x}(t))^* V_{\xi_x(t)}, \qquad V \in T_{\xi_\gamma(t)}\Gamma_X,$$

where the operator

$$(P^J_{\xi_x}(t))^* : T_{\xi_x(t)}X \to T_x X$$

is adjoint (w.r.t. the Riemannian structure of X) of the parallel translation

$$P^J_{\xi_x}(t) : T_x X \to T_{\xi_x(t)}X$$

along $\xi_x(t)$ with potential J. That is, $\eta(t) = P^J_{\xi_x}(t)h$ satisfies the SDE

$$\frac{D}{dt}\eta(t) = J(\eta(t)), \qquad \eta(0) = h, \tag{50}$$

where $\frac{D}{dt}$ is the covariant differentiation along the paths of the process ξ (see [22]). It is known that

$$\|P^J_{\xi_x}(t)\| \le e^{tC},$$

where C is the supremum of the spectrum of $J(x)$. This implies obviously the similar estimate for $\mathbf{P}^J_{\xi_\gamma}$:

$$\|\mathbf{P}^J_{\xi_\gamma}(t)\| \le e^{tC}. \tag{51}$$

Let us define a semigroup $\mathbf{T}^{\mathbf{J}}_1(t)$ associated with the process ξ_γ and potential $\mathbf{J}$.

Definition 5 For $V \in \mathcal{F}\Omega^1$, we set

$$\mathbf{T}^{\mathbf{J}}_1(t)V(\gamma) := \mathsf{E}\,\mathbf{P}^J_{\xi_\gamma}(t)V(\xi_\gamma(t)).$$

Let $T^J_1(t)$ be the semigroup acting in $L^2_\sigma\Omega^1(X)$ as

$$T^J_1(t)\nu(x) := \mathsf{E}\,P^J_{\xi_x}(t)^*\nu(\xi_x(t)).$$

The following result describes the structure and properties of the semigroup $\mathbf{T}^{\mathbf{J}}_1(t)$.

Proposition 6 1) $\mathbf{T}^{\mathbf{J}}_1(t)$ *satisfies the estimate*

$$\|\mathbf{T}^{\mathbf{J}}_1(t)V(\gamma)\|_\gamma \le e^{tC}\mathbf{T}_0(t)\|V(\gamma)\|_\gamma. \tag{52}$$

2) *Under the action of the isomorphism* I^1, $\mathbf{T}^{\mathbf{J}}_1(t)$ *obtains the following form*:

$$I^1\mathbf{T}^{\mathbf{J}}_1(t) = \mathbf{T}_0(t)\otimes T^J_1(t)\; I^1. \tag{53}$$

3) $\mathbf{T}^{\mathbf{J}}_1(t)$ *extends to a strongly continuous semigroup in* $L^2_{\pi_\sigma}\Omega^1$.

Proof. 1) The result follows from formula (51).

2) Let $V \in \mathcal{D}\Omega^1$ be given by (13). By the definition of $\mathbf{T}_1^{\mathbf{J}}(t)$ and the construction of the process ξ_γ, we have

$$\mathbf{T}_1^{\mathbf{J}}(t)V(\gamma) = \mathsf{E}\,\mathbf{P}_{\xi_\gamma}^{\mathbf{J}}(t)V(\xi_\gamma(t))$$

and

$$\begin{aligned}(\mathbf{T}_1^{\mathbf{J}}(t)V(\gamma))_x &= \mathsf{E}\,F(\xi_\gamma(t)\setminus\{\xi_x(t)\})P_{\xi_x}^{J_1}(t)^*\nu(\xi_x(t)) \\ &= \mathsf{E}\,F(\xi_\gamma(t)\setminus\{\xi_x(t)\})\,\mathsf{E}_{\xi_x}\,P_{\xi_x}^{J_1}(t)^*\nu(\xi_x(t)) \\ &= \mathbf{T}_0(t)F(\gamma\setminus\{x\})T_1^J(t)\nu(x),\end{aligned}$$

E_{ξ_x} meaning the expectation w.r.t. the process $\xi_x(t)$, from where the result follows.

3) The result follows from the corresponding results for semigroups $\mathbf{T}_0(t)$ and $T_1^J(t)$, which are well-known (see [7] resp. [22]). ■

Let $\mathbf{H}_1^{\mathbf{J}}$ and H_1^J be the generators of $\mathbf{T}_1^{\mathbf{J}}(t)$ and $T_1^J(t)$, respectively.

Now we give probabilistic representations of the semigroups $T_{\pi_\sigma}^B(t)$ and $T_{\pi_\sigma}^R(t)$ associated with operators $H_{\pi_\sigma}^B$ and $H_{\pi_\sigma}^R$, respectively. We set $J_0 = 0$, $J_1(x) = R_\sigma(x)$ (cf. (20)). Let us remark that $P_{\xi_x}^{J_0}(t) \equiv P_{\xi_x}(t)$ is the parallel translation of 1-forms along the path ξ_x, and we have $H_1^{J_0} = -H_\sigma^B$ and $H_1^{J_1} = -H_\sigma^R$ on $\Omega_0^1(X)$. We have the following

Theorem 6 1) *For $W \in \mathcal{D}\Omega^1$, we have*

$$H_{\pi_\sigma}^B W = -\mathbf{H}_1^{\mathbf{J}_0}W, \qquad H_{\pi_\sigma}^R W = -\mathbf{H}_1^{\mathbf{J}_1}W. \tag{54}$$

2) *As L^2-semigroups,*

$$T_{\pi_\sigma}^B(t) = \mathbf{T}_1^{\mathbf{J}_0}(t), \qquad T_{\pi_\sigma}^R(t) = \mathbf{T}_1^{\mathbf{J}_1}(t). \tag{55}$$

3) *The semigroups $T_{\pi_\sigma}^B(t)$ and $T_{\pi_\sigma}^R(t)$ satisfy the estimates:*

$$\|T_{\pi_\sigma}^B(t)V(\gamma)\|_\gamma \le \mathbf{T}_0(t)\|V(\gamma)\|_\gamma$$

and

$$\|T_{\pi_\sigma}^R(t)V(\gamma)\|_\gamma \le e^{tC}\mathbf{T}_0(t)\|V(\gamma)\|_\gamma.$$

Proof. 1) It follows directly from the decomposition (53) that, on $\mathcal{D}\Omega^1$, we have

$$I^1\,\mathbf{H}_1^{\mathbf{J}} = \left(\mathbf{H}_0 \boxplus H_1^J\right) I^1. \tag{56}$$

Setting $J = J_0$ and $J = J_1$ and comparing (27) and (56), we obtain the result.

2) The statement follows from (54) and the essential self-adjointness of $H_{\pi_\sigma}^B$ and $H_{\pi_\sigma}^R$ on $\mathcal{D}\Omega^1$ by applying Proposition 6, 3), with $J = J_0$ and $J = J_1$, respectively.

3) The result follows from (55) and (52). ■

5 Acknowledgments

The first author is very grateful to the organizers for giving him the possibility to present his results at a most stimulating conference. It is a great pleasure to thank our friends and colleagues Yuri Kondratiev, Tobias Kuna, and Michael Röckner for their interest in this work and the joy of collaboration. We are also grateful to V. Liebscher for a useful discussion. The financial support of SFB 256 and DFG Research Project AL 214/9-3 is gratefully acknowledged.

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